Concerning S. Singh and N. Sharma: "Comment on 'Paradox in the classical treatment of the Stern–Gerlach experiment'"

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in the z direction in the region of the beam. Then although the inhomogeneity causes, on an atom, a transverse force along with an equal force in the field direction, the precession of magnetic moment averages out the transverse component justifying the usual classical treatment described in the introductory textbooks.

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[a] On leave from Department of Physics, Government College, Ajmer.

[1] P. Alstrøm, P. Hjorth, and R. Mattuck, Am. J. Phys. **50**, 697 (1982).

[2] Similar remarks are made without elaboration in a few textbooks, e.g., D. Bohm, *Quantum Theory* (Prentice-Hall, New York, 1951), p. 594; A. K. Ghatak and S. Lokanathan, *Quantum Mechanics* (The Macmillan India, Delhi, 1977), p. 427.

[3] A. Böhm, *Quantum Mechanics* (Springer-Verlag, New York, 1979). An analysis, covering all the relevant points, is given in Sec. XIII.A, p. 303.

[4] See Ref. 1, Fig. 1, p. 697 for the choice of coordinates x, y, and z.

[5] $\partial_x B_x$ is not necessarily zero at $x = 0$.

[6] See, for example, J. B. Taylor, Phys. Rev. **28**, 576 (1926); and N. F. Ramsey, *Molecular Beams* (Clarendon, Oxford, 1956), p. 100.

[7] See Ref. 1, p. 697.

[8] J. B. Taylor, Ref. 6, p. 577.

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In an earlier note[1] it was mentioned that most textbook authors assume that the force on a magnetic moment sent into a Stern–Gerlach apparatus is exclusively in the direction of the magnetic field. This was in the case where the magnetic field $\mathbf{B}$ and the gradient of $\mathbf{B}\cdot\mathbf{e}_B = B$ were in the same direction as in the middle of the Stern–Gerlach apparatus, see Fig. 1. However, it was shown that only the time average of the force is in the magnetic field direction. Nevertheless, because $1/\omega\tau \simeq 10^{-7}$ ($\omega = eB/mc, \tau = 1/v$) in a typical experiment, where $B \simeq 10^4$ G, the length of the magnet $l \simeq 3$ cm, and the velocity of the atom $v = \sqrt{3.5RT/M} \simeq 6\times 10^4$ cm/s (see Ref. 2), the classical result is the same, namely a line distribution in the $\mathbf{B}$ direction.

Singh and Sharma[3] extend the investigation to the general case, where the beam has a width, and find a formula for the time average of the force on a magnetic moment $\boldsymbol{\mu}$, but not in the often used atom system of coordinates with the origin in the atom, the y axis in the direction of motion, and the z axis parallel to the magnetic field. Instead they describe the situation in the coordinates shown in Fig. 1. Their expression for the average force can, after some manipulations, be reduced to

$$\mathbf{F}_{\text{av}} = \boldsymbol{\mu}(0)\cdot\mathbf{e}_B \nabla B, \tag{1}$$

which is the textbook result.[4] If we use

$$\nabla B = (\mathbf{e}_B\cdot\nabla)\mathbf{B},$$

and note that the vector product is conserved under transformations of coordinates, we get in the atom system

$$\mathbf{F}_{\text{av}} = \boldsymbol{\mu}_z(0)\frac{\partial}{\partial z}\mathbf{B}. \tag{2}$$

This is exactly what one should expect if we remind ourselves of the circumstance that the magnetic moment of the atom is precessing around the magnetic field direction. This consideration and the result (2) was already given by Stern[5] in 1921. In the light of this result he proposed that the edge, in the now well-known Stern–Gerlach apparatus, produces the inhomogeneous field.

Singh and Sharma suggest that the homogeneous part $B_0\mathbf{k}$ of the magnetic field in the coordinates shown in Fig. 1 has to be great in proportion to the inhomogeneous part $\mathbf{B}_1(x,z)$. This condition will, in the region near the symmetry plane where $|\partial B_z/\partial x| \ll |\partial B_z/\partial z|$, give an angle φ between $\mathbf{F}_{\text{av}}$ and $\mathbf{k}$

$$\varphi \simeq \frac{\partial B_z/\partial x}{\partial B_z/\partial z}.$$

However, $\partial B_z/\partial x$ and $\partial B_z/\partial z$ change with the distance from the symmetric y–z plane, and we will not get a homogeneous distribution, see, e.g., Fig. 2; in fact, because B_x is zero at $x = 0$ and for great values of x, there will be points where $\partial B_x/\partial x$ and therefore $\partial B_z/\partial z$ vanish.

It is of importance to point out that in the equation

$$\dot{\boldsymbol{\mu}} = \boldsymbol{\omega}\times\boldsymbol{\mu},$$

the angular velocity $\boldsymbol{\omega}$ is equal to $(e/mc)\mathbf{B}(x_0,z_0)$, where

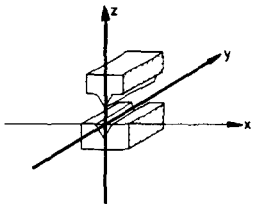

Fig. 1. Coordinates in the Stern–Gerlach experiment.

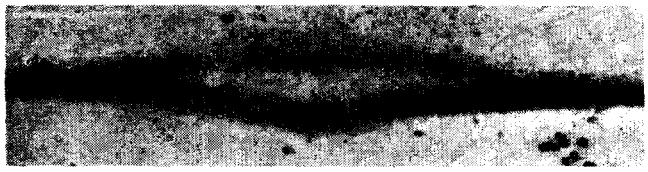

Fig. 2. Image obtained by J. B. Taylor, see Phys. Rev. **28**, 581 (1926).

$(x_0,0,z_0)$ is the point where the atom enters the field. Therefore the proper value of $\mathbf{F}$ is found from

$$\mathbf{F} = \mu_x \nabla B_x + \mu_z \nabla B_z \text{ or } \mathbf{F} = (\boldsymbol{\mu}\cdot\nabla)\mathbf{B}. \quad (3)$$

This formula was used in Ref. 1 as well as in Ref. 3. The usual textbook error is to set $|\boldsymbol{\mu}|\cos\theta = \boldsymbol{\mu}(t)\cdot\mathbf{e}_B(x,y)$ equal to $\boldsymbol{\mu}(t)\cdot\mathbf{e}_{B(x_0,z_0)} = \boldsymbol{\mu}(0)\cdot\mathbf{e}_{B(x_0,y_0)}$, which leads to the expression for $\mathbf{F}$ given by the right side of Eq. (1).

[1]P. Alstrøm, P. Hjort, and R. Mattuck, Am. J. Phys. **50**, 697 (1982).
[2]The factor 3.5 comes from measurements of Stern, see O. Stern, Z. Phys. **2**, 49 (1920).
[3]S. Singh and N. K. Sharma, Am. J. Phys. **52**, 274 (1984).
[4]See, e.g., E. Merzbacher, *Quantum Mechanics* (Wiley, New York, 1970), p. 252.
[5]O. Stern, Z. Phys. **7**, 249 (1921), see p. 251.

A remark on principal-value integrals

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Theorem.[1] Let $f(z)$ be analytic inside and on a contour C except for a finite number of poles a_k inside C and *simple* poles b_k on C, with respective residues $R(a_k)$ and $R(b_k)$. Then

$$PV\int_C f(z)dz = 2\pi i\left[\sum R(a_k) + \tfrac{1}{2}\sum R(b_k)\theta_k/\pi\right],$$

where θ_k is the interior angle of the contour at b_k.

In other words, one can correctly integrate through a simple pole at the expense of counting only half the residue when the contour does not have a corner at b_k, a quarter of the residue at a right angle, etc., exactly as intuition would suggest.

It is our experience that many people react with horror to this formulation, but we do not understand why. In fact, the theorem is easily proved by indenting the contour by small arcs of circles centered at the b_k, just as is always done in each special case.[2] It saves considerable effort if one can apply a general result instead: we do not have to build a calculator each time we want to multiply two numbers.[3]

[1]The theorem for $\theta = \pi$ can be found, for example, in E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (Cambridge U. P., New York, 1927), p. 177; D. S. Mitrinović and J. D. Kečkić, *Cauchyjev Račun Ostataka sa Primenama* (Naučna Knjiga, Belgrade, 1978), p. 150; M. L. Boas, *Mathematical Methods in the Physical Sciences* (Wiley, New York, 1983), pp. 605,611. The general result is implicit in a number of places, for example, P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953), Vol. 1, p. 368, where the authors inadvertently give the impression that $\theta = \pi$ always; G. F. Carrier, M. Krook, and C. E. Pearson, *Functions of a Complex Variable* (McGraw-Hill, New York, 1966), p. 39; but we have not been able to locate an explicit statement of the general case.
[2]For example, R. P. Leavitt and C. A. Morrison, Am. J. Phys. **50**, 1112 (1982).
[3]"These theorems once established—and their proofs involve but slight formal work, rather an appreciation of the inner content of the situation—it is the part of wisdom to work with these *results*, not to *prove* them afresh every time one needs them. We do not compute our logarithms each time, before we use them."—W. F. Osgood, *Advanced Calculus* (Macmillan, New York, 1929), p. 308.

Comment on "Least squares when both variables have uncertainties"

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Jay Orear[1] deserves commendation for reiterating the need to use a more general curve fitting method than standard least squares in the case when the measured values of both variables in a functional relation between x and y contain uncertainty. His assertions that the effective variance method[2] often gives a much better solution and is easy to apply are also correct. Unfortunately, one statement in his article is misleading.

That statement is "...it is always exact for linear regression" (in reference to the effective variance method), where "linear regression" refers to fits of functions linear in x. The equation for the weighted sum of squares of residuals S, Orear's Eq. (6), is the correct one to minimize in the case of linear regression, and a good approximation in most other cases. But it is important to clarify what is meant by "the effective variance method." A method that employs a stan-